Probabilities for specific Texas Hold'em board textures

Board texture	Probability	Formula
Flop		
The flop contains a pair	**17.2%** (1 : 4.8)	$1-\binom{13}{3}\times 4^3\times\binom{52}{3}^{-1}$
The flop contains trips	**0.235%** (1 : 424)	$13\times 4\times\binom{52}{3}^{-1}$
The flop is single-suited	**5.18%** (1 : 18)	$\binom{13}{3}\times 4\times\binom{52}{3}^{-1}$
The flop contains two different suits	**55.1%** (1.2 : 1)	$1-\left(4\times 13^3+\binom{13}{3}\times 4\right)\times\binom{52}{3}^{-1}$
The flop contains three different suits (rainbow flop)	**39.8%** (1 : 1.5)	$4\times 13^3\times\binom{52}{3}^{-1}$
The flop is single coloured (all black or all red)	**23.5%** (1 : 3.3)	$2\times\binom{26}{3}\times\binom{52}{3}^{-1}$
The flop contains at least one ace (or any other specific rank)	**21.7%** (1 : 3.6)	$1-\binom{48}{3}\times\binom{52}{3}^{-1}$
The flop contains at least one ace or king (or any two other specific ranks)	**40.1%** (1 : 1.5)	$1-\binom{44}{3}\times\binom{52}{3}^{-1}$
The flop contains the **A♠** (or any other specific card)	**5.77%** (1 : 16)	$1-\binom{51}{3}\times\binom{52}{3}^{-1}$
Flop and Turn		
The board contains a pair	**32.4%** (1 : 2.1)	$1-\binom{13}{4}\times 4^4\times\binom{52}{4}^{-1}$
The board contains trips	**0.922%** (1 : 107)	$13\times 4\times 48\times\binom{52}{4}^{-1}$
The board contains quads	**0.004802%** (1 : 20,824)	$13\times\binom{52}{4}^{-1}$
The board is single-suited	**1.06%** (1 : 94)	$\binom{13}{4}\times 4\times\binom{52}{4}^{-1}$
The board contains three cards of the same suit	**16.5%** (1 : 5.1)	$4\times\binom{13}{3}\times 39\times\binom{52}{4}^{-1}$
The board contains two cards of the same suit	**71.9%** (2.6 : 1)	-
The board contains four different suits (rainbow board)	**10.5%** (1 : 8.5)	$13^4\times\binom{52}{4}^{-1}$
The board is single coloured (all black or all red)	**11.0%** (1 : 8.1)	$2\times\binom{26}{4}\times\binom{52}{4}^{-1}$
Full board (flop, turn and river)		
The board contains a pair	**49.3%** (1 : 1.0)	$1-\binom{13}{5}\times 4^5\times\binom{52}{5}^{-1}$
The board is single-suited	**0.198%** (1 : 504)	$\binom{13}{5}\times 4\times\binom{52}{5}^{-1}$
The board is single coloured (all black or all red)	**5.06%** (1 : 19)	$2\times\binom{26}{5}\times\binom{52}{5}^{-1}$

All probabilities in this table are assuming you don't know anything about the 52 cards (e.g. have not seen your hole cards).